AD-A157 868

NSWC TR 85-17

(2)

DIFFERENTIAL CORRECTION OF ORBITS BY KEPLER VERSUS CARTESIAN PARAMETERS

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STRATEGIC SYSTEMS DEPARTMENT

DECEMBER 1984

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85 8 02 040

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NSWC TR 85-17	2. GOVT ACCESSION NO. AD-A157 868	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) DIFFERENTIAL CORRECTION OF ORBITS BY KEPLER VERSUS CARTESIAN PARAMETERS		5. TYPE OF REPORT & PERIOD COVERED FINAL
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) C. Harris Seay		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center Dahlgren, VA 22448-5000		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NIF
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Mapping Agency Headquarters Washington, DC 20305		12. REPORT DATE December 1984
		13. NUMBER OF PAGES 17
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Orbit Determination
Coordinate Systems
Orbital Parameters

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Numerical integration of orbits of near earth satellites is usually carried out in a Cartesian coordinate system. When performing the differential correction of an initial orbit to match observations, the parameters of the solution are sometimes the corrections to the position and velocity vector of the satellite at epoch. At the Naval Surface Weapons Center, however, the parameters chosen are Kepler parameters modified to avoid singularity for circular orbits. The rate of convergence for a sample case was not found to

be significantly different for the two parameterizations, but a significant gain in numerical accuracy was obtained with the set of Kepler parameters.

FOREWORD

Parameter improvement of near earth satellite orbits often use the position and velocity vector as parameters but Keplerian elements are an alternative set of parameters. The rate of convergence of the two sets and the loss of significant digits in solution are presented in this report.

This work was accomplished in the Space and Ocean Geodesy Branch, Space and Surface Systems Division. The report was reviewed by Dr. R. J. Anderle, Research Associate of the Strategic Systems Department.

Approved by:

THOMAS A. CLARE, Head
Strategic Systems Department

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INTRODUCTION

Numerical integration of orbits of near earth satellites is usually carried out in a Cartesian coordinate system. When performing the differential correction of an initial orbit to match observations, the parameters of the solution are sometimes the corrections to the position and velocity vector of the satellite at epoch. At the Naval Surface Weapons Center, however, the parameters chosen are Kepler parameters modified to avoid singularity for circular orbits. This report compares the rate of convergence of this orbit and the number of digits lost in the computation for the two parameter sets for a sample orbit computation.

PROCEDURE

The data used in this study were 2 days of Doppler data from the "HILAT" satellite which is in a nearly circular orbit at an 800 km altitude and at an inclination of 82.2°. CELEST,* a large scale orbit determination program, was used to compute orbits and solve for parameters. The parameters solved for were six orbit constants, one drag coefficient, one pair of resonant gravity coefficients ($C_{14,14}$ and $S_{14,14}$), and two pole position parameters. Two parameters were also included for each pass to correct for deviations from nominal tropospheric refraction corrections, which were assigned an accuracy of 10 percent, and for errors in the relative frequencies of the satellite and ground station oscillators. A converged orbit was computed and labeled as the "standard." The Keplerian semi-major axis (A) of the standard was perturbed by 10 m and corresponding perturbations were made to the Cartesian parameters. Both sets of perturbed parameters were fit to the Doppler data for one iteration. The resultant orbits were compared to the "standard" orbit and to each other. A similar procedure was followed after perturbing the Cartesian X parameter by 10 m and the corresponding Keplerian parameters by the equivalent values. Also computed were the number of significant digits lost in solution by computing the product of the diagonal elements of the normal equation matrix and the corresponding diagonal element of the inverse matrix.

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*O'Toole, J. W., CELEST Computer Program for Computing Satellite Orbits, NSWC TR 3565, Oct 1976.

RESULTS

The orbit was perturbed, primarily secularly, to about 2600 m after 2 days (Figure 1) for the perturbed semi-major axis case and to about 1450 m for the perturbed X case. After a single iteration the perturbed orbit agreed with the converged orbit to about 5 m for the perturbed A case and to about 1.5 m for the perturbed X case. For this set of data there was only a marginal difference between the use of Cartesian or Keplerian elements in the solution, with the Keplerian being a little better when X was perturbed and the Cartesian slightly better when A was perturbed (Table 1). For analysis the differences between the orbits are decomposed into three components: A component tangential to the velocity vector, a radial component, and an out of plane component. All three components are shown graphically for one case in Figures 2-4. For the other cases only the tangential component (Figures 5-7) is given graphically because the radial and out of plane components are short term periodic as seen in the first case; the amplitude of the periodic error in those components is given by the "peak error" in Table 1. Table 1 gives the peak differences and the rms differences for each case broken down into its three components and the total difference. It can be seen in Table 2, which shows the number of significant digits lost in solution, that using Keplerian elements in solution results in about four fewer significant digits lost in solution for the six orbit parameters; for the other parameters (drag, gravity, polar motion), there is no difference in the number of digits lost between the two representations of the parameters.

CONCLUSION

Whether Keplerian or Cartesian parameters are used in solution does not affect the rate of convergence of fits to 2 days of data, but using Keplerian parameters does reduce the number of significant digits lost in the solution. This loss of significant digits could be of importance, for example, in gravity solutions where the loss of significance in the orbit portion of the matrix will transfer into arc eliminated matrices of different satellites which are combined in a solution for gravity parameters.

The rate of convergence may be different for longer spans of data which were not included in the scope of this report.

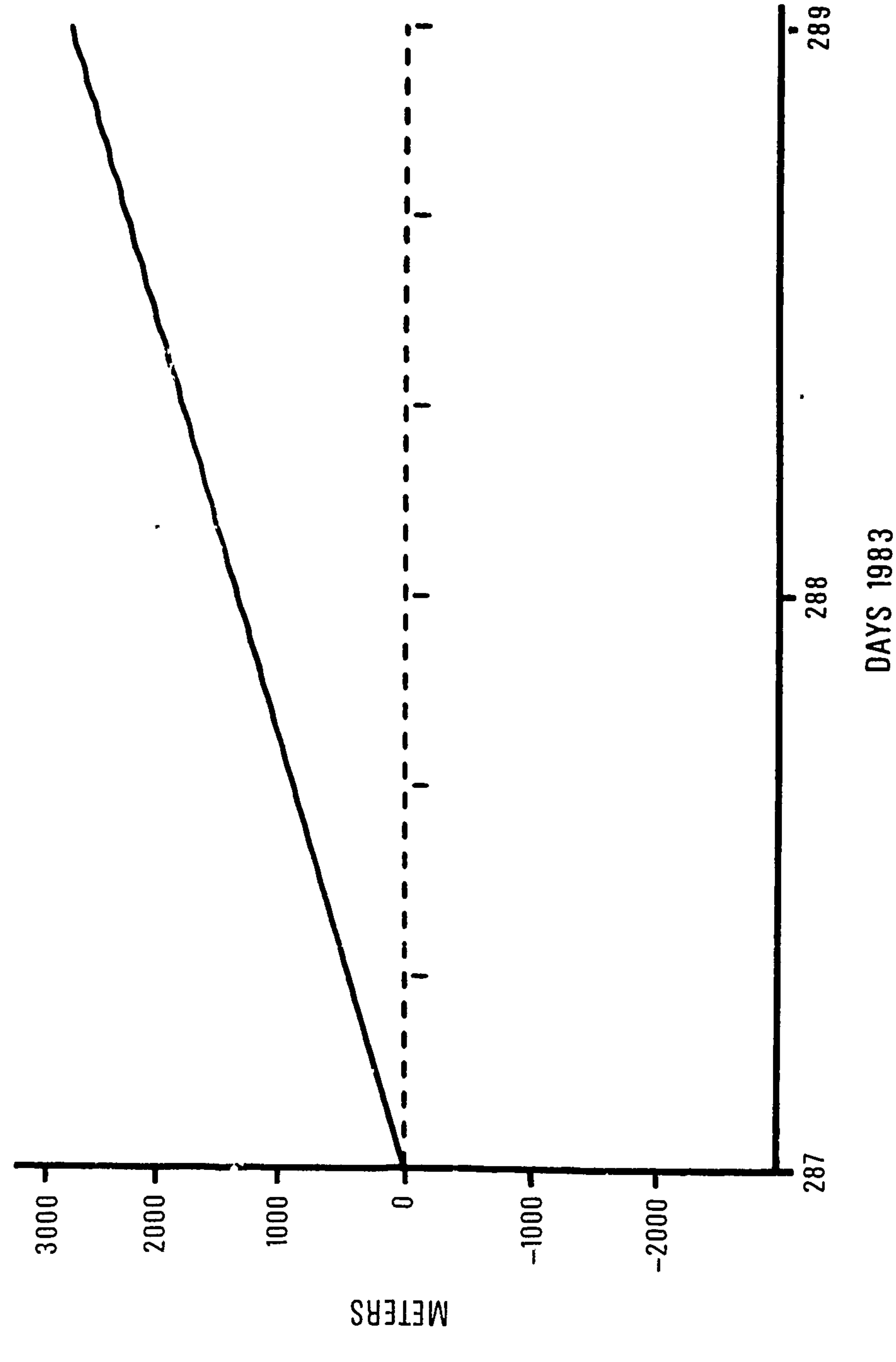

FIGURE 1. EFFECT OF PERTURBED A ON TANGENTIAL COMPONENT

TABLE 1. ORBIT DIFFERENCES (m)

CARTESIAN VERSUS KEPLERIAN PARAMETERS

PERTURB :	A	A	X	X	
SOLVE FOR :	KEPLERIAN	CARTESIAN	KEPLERIAN	CARTESIAN	
TOTAL	5.40	4.48	1.19	1.69	PEAK
	2.37	1.87	.48	1.02	RMS
RADIAL	1.59	1.05	.32	.32	PEAK
	.50	.45	.16	.17	RMS
TANGENTIAL	4.79	4.10	1.19	1.38	PEAK
	1.25	1.32	.40	.49	RMS
OUT OF PLANE	2.83	1.83	.28	1.29	PEAK
	1.94	1.25	.19	.88	RMS

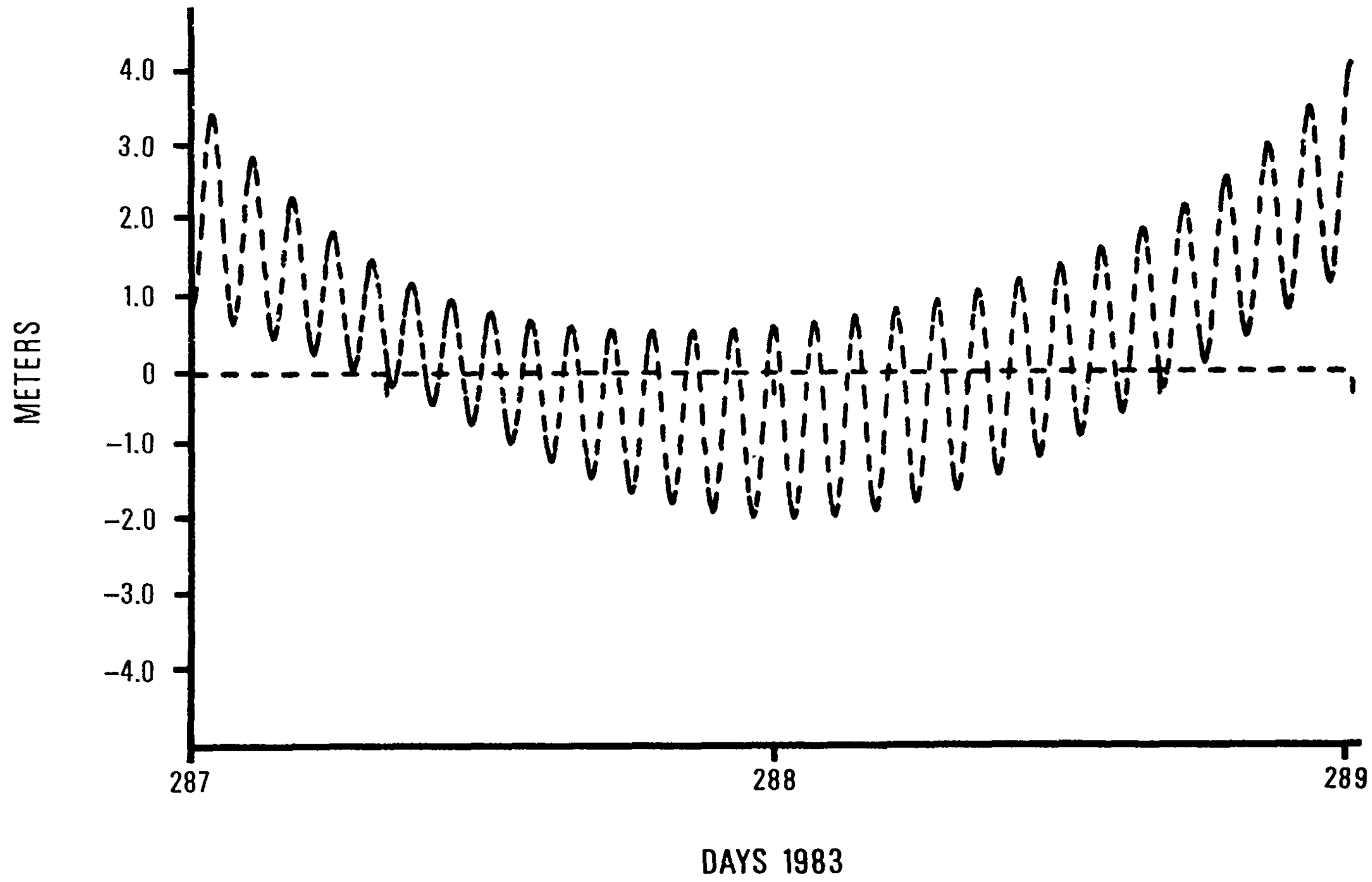

FIGURE 2. ORBIT DIFFERENCE IN TANGENTIAL COMPONENT DUE TO PERTURBED A, CARTESIAN--STANDARD

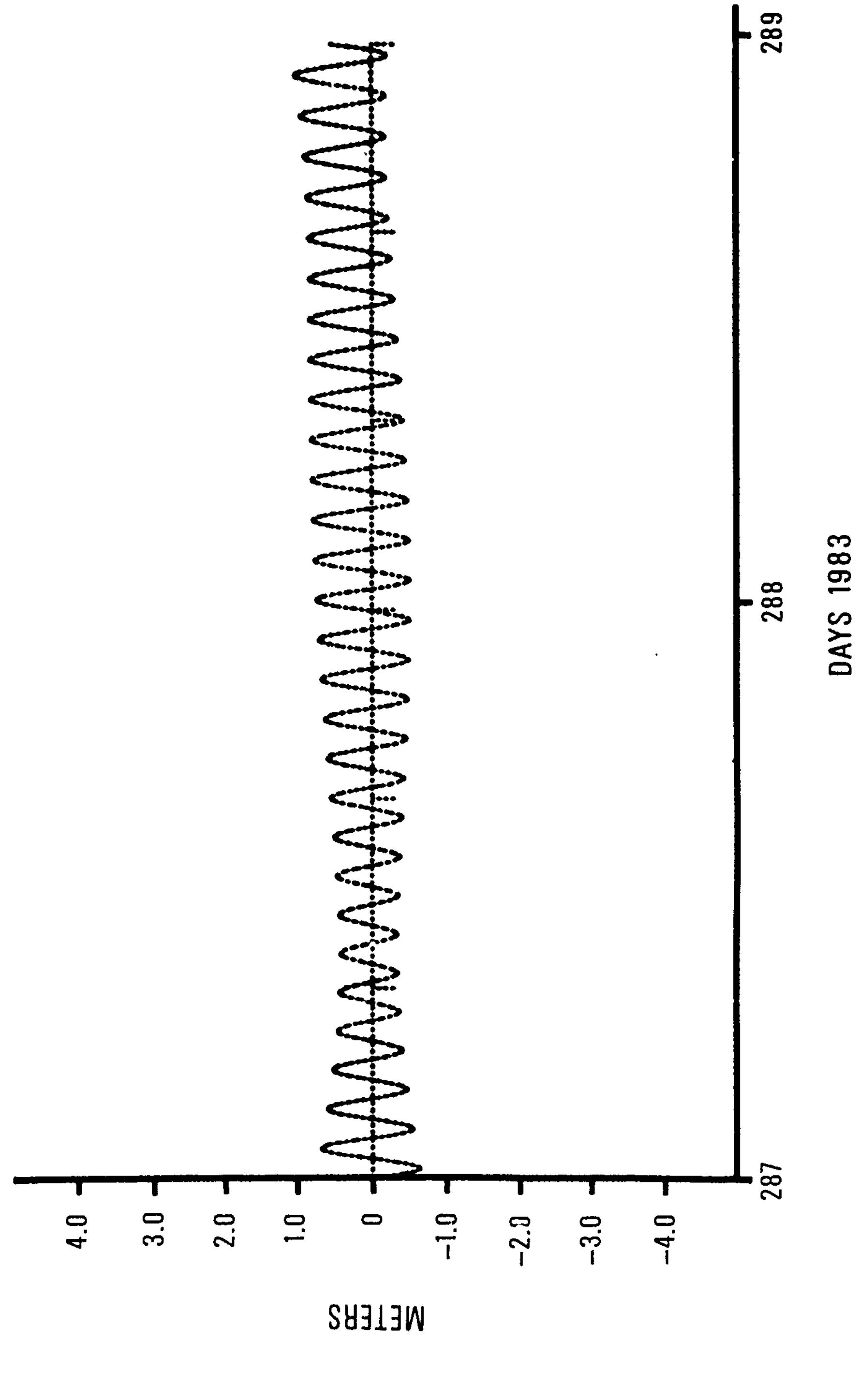

FIGURE 3. ORBIT DIFFERENCE IN RADIAL COMPONENT DUE TO PERTURBED A, CARTESIAN--STANDARD

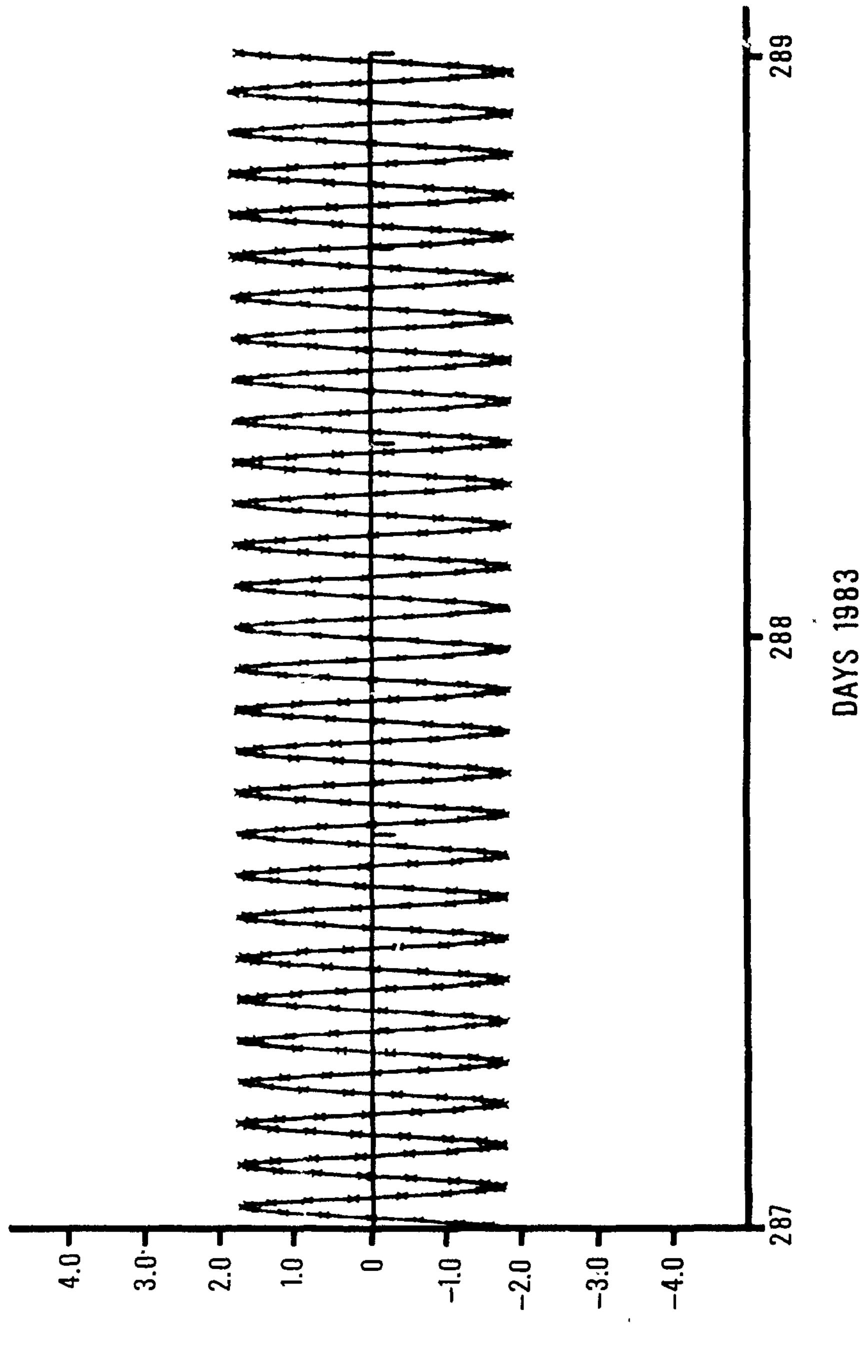

FIGURE 4. ORBIT DIFFERENCE IN OUT OF PLANE COMPONENT DUE TO PERTURBED A, CARTESIAN--STANDARD

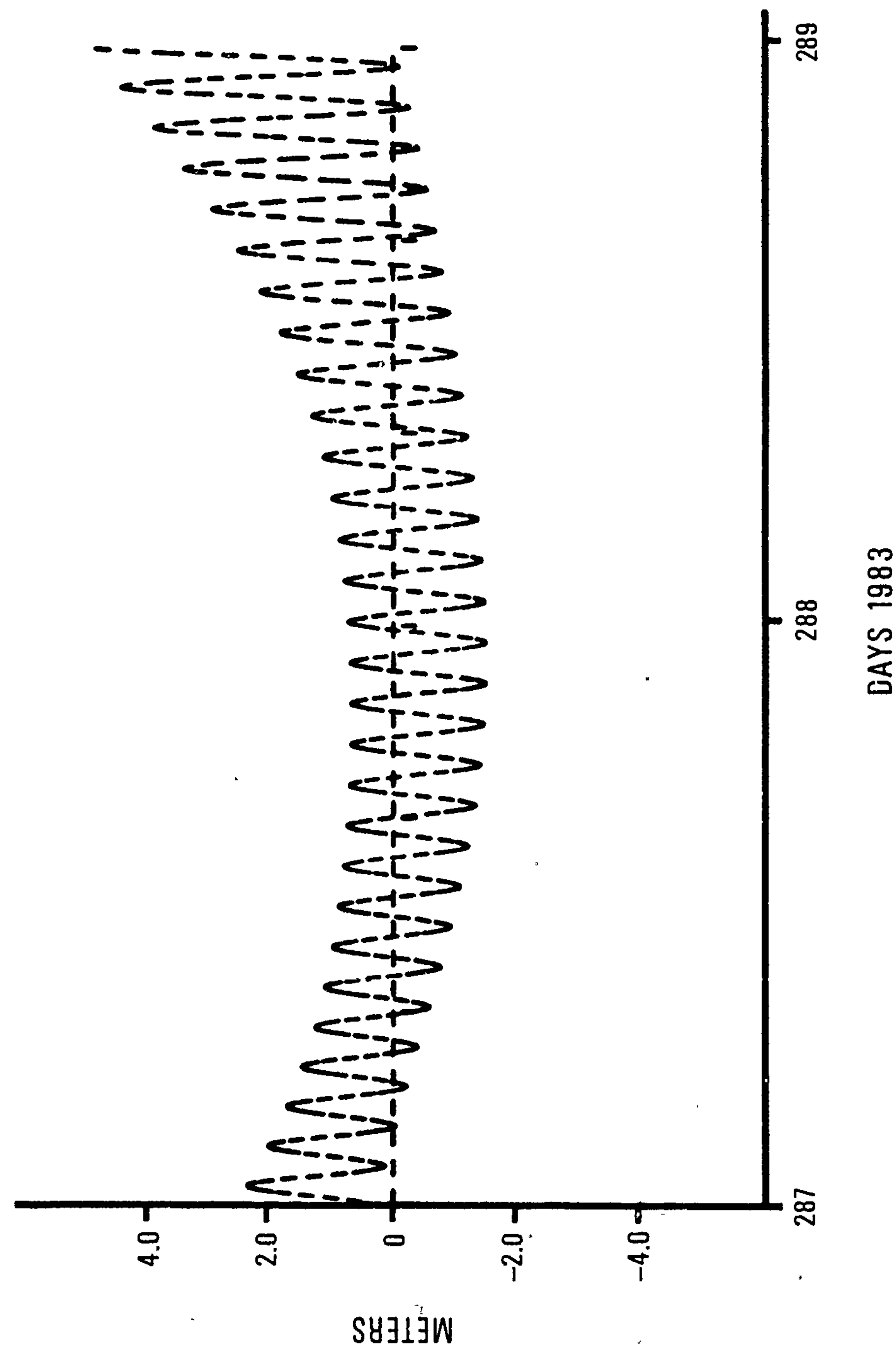

FIGURE 5. ORBIT DIFFERENCE IN TANGENTIAL COMPONENT DUE TO PERTURBED A, KEPLERIAN--STANDARD

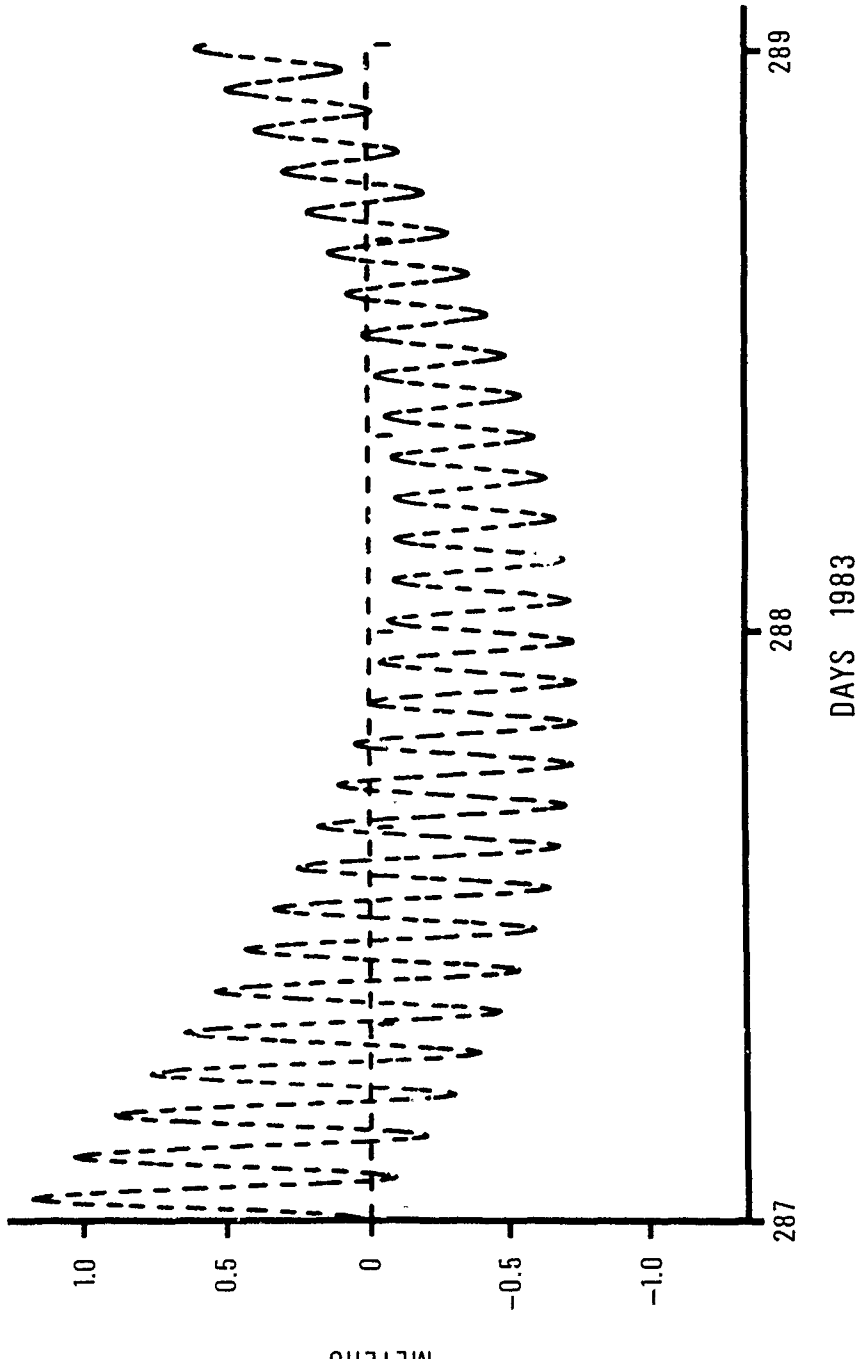

FIGURE 6. ORBIT DIFFERENCE IN TANGENTIAL COMPONENT DUE TO PERTURBED X, KEPLERIAN--STANDARD

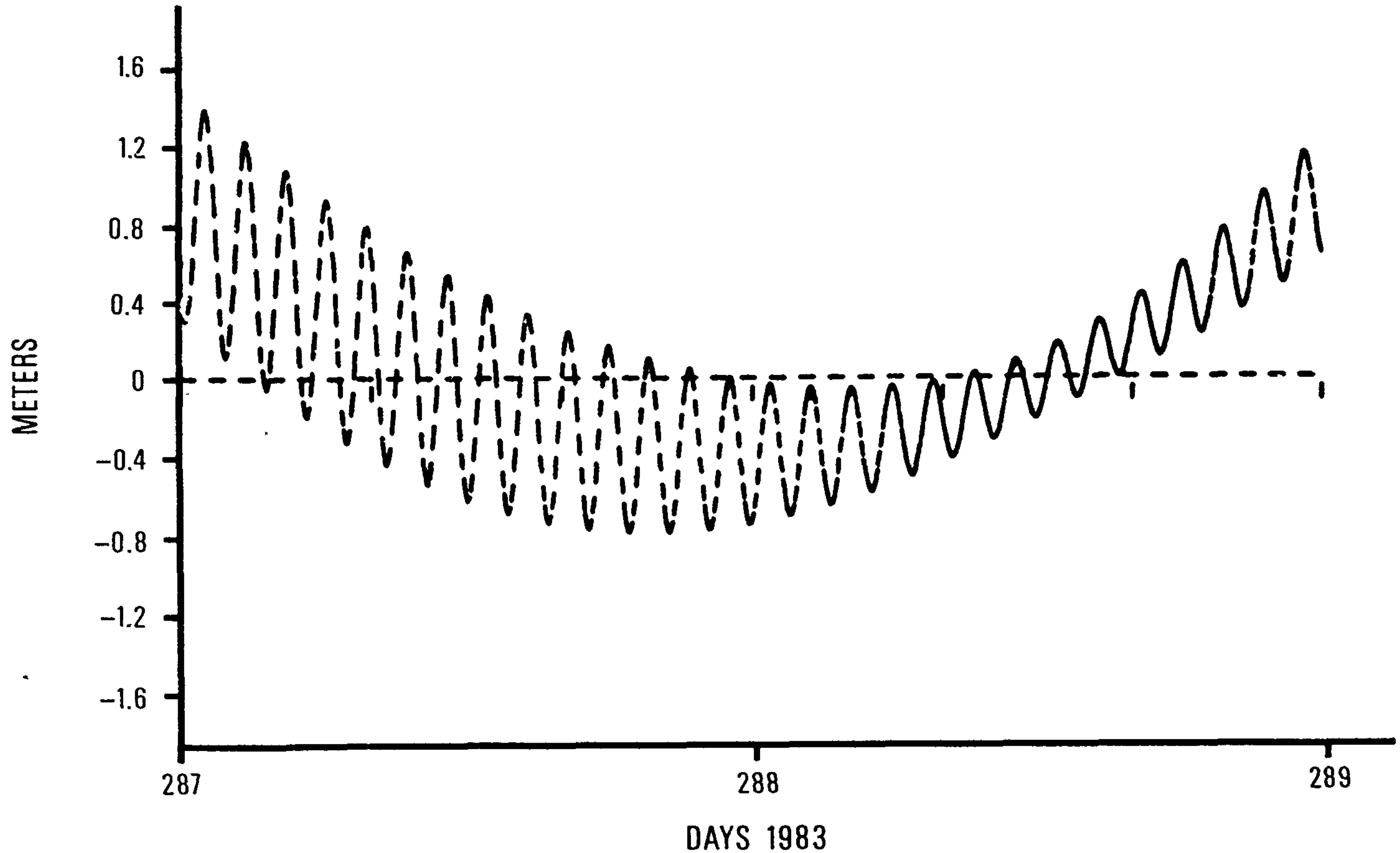

FIGURE 7. ORBIT DIFFERENCE IN TANGENTIAL COMPONENT DUE TO PERTURBED X, CARTESIAN--STANDARD

TABLE 2. PRODUCT OF DIAGONAL ELEMENTS OF THE NORMAL MATRIX AND ITS INVERSE

			PERTURB A		PERTURB X	
KEPLERIAN PARAMETER	CARTESIAN PARAMETER	"STANDARD"	KEPLERIAN PARAMETERS	CARTESIAN PARAMETERS	KEPLERIAN PARAMETERS	CARTESIAN PARAMETERS
A	X	150	110	47263	139	49216
e sin ω	Y	14	13	240906	13	253139
e cos ω	Z	1	1	368714	1	399011
I	$\dot{X}$	1	1	76180	1	81422
M + ω	$\dot{Y}$	30	29	187658	29	201202
Ω	$\dot{Z}$	1	1	341639	1	368762
DRAG		64	57	57	58	57
$C_{14,14}$		15	15	14	15	15
$S_{14,14}$		8	7	7	7	8
Δq		1	1	1	1	1
Δp		1	1	1	1	1

A semi-major axis
e eccentricity
I inclination
M mean anomaly
ω argument of perigee
Ω right ascension of ascending node

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